

Distributed Deterministic Broadcasting in Uniform-Power Ad Hoc Wireless Networks

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Abstract

Development of many futuristic technologies, such as MANET, VANET, iThings, nano-devices, depend on efficient distributed communication protocols in multi-hop ad hoc networks. A vast majority of research in this area focus on design heuristic protocols, and analyze their performance by simulations on networks generated randomly or obtained in practical measurements of some (usually small-size) wireless networks. Moreover, they often assume access to truly random sources, which is often not reasonable in case of wireless devices. In this work we use a formal framework to study the problem of broadcasting and its time complexity in *any* two dimensional Euclidean wireless network with uniform transmission powers. For the analysis, we consider two popular models of ad hoc networks based on the Signal-to-Interference-and-Noise Ratio (SINR): one with opportunistic links, and the other with randomly disturbed SINR. In the former model, we show that one of our algorithms accomplishes broadcasting in $O(D \log^2 n)$ rounds, where n is the number of nodes and D is the diameter of the network. If nodes know a priori the granularity g of the network, i.e., the inverse of the maximum transmission range over the minimum distance between any two stations, a modification of this algorithm accomplishes broadcasting in $O(D \log g)$ rounds. Finally, we modify both algorithms to make them efficient in the latter model with randomly disturbed SINR, with only logarithmic growth of performance. Ours are the first provably efficient and well-scalable, under the two models, distributed deterministic solutions for the broadcast task.

Keywords: Ad hoc wireless networks, Uniform-power networks, Signal-to-Interference-and-Noise-Ratio (SINR) models, Broadcast problem, Local leader election, Distributed algorithms.

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1 Introduction

In this work we consider a broadcast problem in ad-hoc wireless networks under the Signal-to-Interference-and-Noise-Ratio (SINR) models. Wireless network consists of at most n stations, also called nodes, with unique integer IDs and uniform transmission powers P , deployed in the two-dimensional space with Euclidean metric. Each station initially knows only its own ID, location and the upper bound n on the number of nodes.

There are three dominating approaches in the literature to analyze performance of communication protocols in wireless networks based on the Signal-to-Interference-and-Noise-Ratio. The first model is based on the *SINR with random disturbances*: each measured SINR is randomly changed according to some stochastic distribution [20]. This model takes into account all signal disturbances that may occur in the environment, apart of the average background noise and signal deterioration captured by the deterministic SINR formula.

The second model is based on the notion of communication graph, which contains only those communication links that are between stations of distance $(1 - \varepsilon)$ times the maximum transmission range. Even though communication between two nodes not directly connected in such communication graph may occur in practice, it is very unlikely especially if relatively many nodes intend to transmit. Therefore, intuitively, the algorithm may rely only on the links in the communication graph, and in order to be fairly treated, its performance should be analyzed as if only those links were available. For example, the diameter of the communication graph is a natural lower bound on broadcasting problem, even though in practice there might exist shorter paths (but being very difficult to propagate any message along themselves, due to large distances and substantial signal deterioration). We call this setting the *model with opportunistic links*, c.f., [21].

The third model is based on additional assumption that in order to receive a message, not only the SINR must be above some (relatively high) threshold, but also the received dominating signal must be sufficiently large. We call this setting the *SINR model with weak devices*, c.f., [12, 14].

In this work we focus only on the first two models. We consider two settings: one with no local knowledge being provided a priori to the nodes, and the other where each node knows the granularity g of the network, i.e., the inverse of the maximum transmission range over the minimum distance between any two stations.

In the broadcast problem considered in this work, there is one designated node, called a source, which has a piece of information, called a source message or a broadcast message, which must be delivered to all other accessible (not necessarily directly) nodes by using wireless communication. In the beginning, only the source is executing the broadcast protocol, and the other nodes join the execution after receiving the broadcast message for the first time. The goal is to minimize time needed for accomplishing the broadcast task.

1.1 Previous and Related Results

In this work, we study the performance of *distributed deterministic broadcasting* in ad hoc wireless networks under the two SINR-based physical models mentioned above. In what follows, we discuss most relevant results in the SINR-based models, and the state of the art obtained in the older Radio Network model.

SINR models. In the SINR model with opportunistic links, slightly weaker task of *local* broadcasting in ad hoc setting, in which nodes have to inform only their neighbors in the corresponding communication graph, was studied in [22]. The considered setting allowed power control by deterministic algorithms, in which, in order to avoid collisions, stations could transmit with any power smaller than the maximal one. Randomized solutions for contention resolution [14] and local broadcasting [10] were also obtained. Recently, a distributed randomized algorithm for multi-broadcast has been presented [21] for uniform networks. Although the problem solved in that paper is a generalization of broadcast, the presented solution is restricted merely to networks having the communication graph connected for $\varepsilon = \frac{2}{3}r$, where r is the largest possible SINR ratio. In contrast, our solutions are efficient and scalable for *any* networks with communication graph connected for *any* value of $\varepsilon < \frac{1}{2}$. (In case of $\varepsilon \in [1/2, 1)$, one could take our algorithm for $\varepsilon' = 1/3$, which guarantees at least as good asymptotic performance.)

In the SINR model with random disturbances, motivated by many practical works, c.f., [20], we are not aware of any theoretical analysis of distributed deterministic broadcasting problem.

In the model of weak devices, c.f., [14] studying randomized local broadcast, broadcasting algorithms are not scalable (in terms of network diameter), unless nodes know their neighbors in the corresponding communication graph in advance [12]. This is a fundamental difference between the models considered in this paper, which do not impose any additional physical constraints on receiving devices apart of the SINR threshold, and the model of weak devices which cannot decode weak signals. On the positive side, a scalable (in terms of the maximum node degree) distributed deterministic construction of efficient backbone sub-network was showed in [13]. Once such a network is spanned, scalable (in terms of the diameter of the original network) distributed solutions to many communication tasks can be constructed.

There is a vast amount of work on centralized algorithms under the classical SINR models. The most studied problems include connectivity, capacity maximization, link scheduling types of problems; for recent results and references we refer the reader to the survey [11].

Radio network model. There are several papers analyzing broadcasting in the radio model of wireless networks, under which a message is successfully heard if there are no other simultaneous transmissions from the *neighbors* of the receiver in the communication graph. This model does not take into account the real strength of the received signals, and also the signals from outside of some close proximity. In the *geometric* ad hoc setting, Dessmark and Pelc [5] were the first who studied the broadcast problem. They analyzed the impact of local knowledge, defined as a range within which stations can discover the nearby stations. Emek et al. [6] designed a broadcast algorithm working in time $O(Dg)$ in UDG radio networks with eccentricity D and granularity g , where eccentricity was defined as the minimum number of hops to propagate the broadcast message throughout the whole network. Later, Emek et al. [7] developed a matching lower bound $\Omega(Dg)$. Mobility aspects of communication were studied in [8]. There were several works analyzing deterministic broadcasting in geometric graphs in the centralized radio setting, c.f., [9, 19].

The problem of broadcasting is well-studied in the setting of *graph radio model*, in which stations are not necessarily deployed in a metric space; here we restrict to only the most relevant results. In deterministic ad hoc setting with no local knowledge, the fastest $O(n \log(n/D))$ -time algorithm in symmetric networks was developed by Kowalski [15], and almost matching lower bound was given by Kowalski and Pelc [16]. For recent results and references in less related settings we refer the reader to [4, 16, 1]. There is also a vast literature on randomized algorithms for broadcasting in graph radio model [17, 16, 3].

1.2 Our Results

In this paper we present distributed deterministic algorithms for broadcasting in ad hoc wireless networks of uniform transmission power, deployed in two-dimensional Euclidean space. The time performance of these protocols is measured in two SINR-based models: with opportunistic links and with random disturbances.

In the former model, when no knowledge of the network topology is a priori provided to the nodes, except of the upper bound n on the number of nodes, one of our algorithms works in $O(D \log^2 n)$ rounds. A variation of this protocol accomplishes broadcast in $O(D \log g)$ rounds in case when nodes know the network granularity before the computation. (It is sufficient that only the source a priori knows network granularity.)

We show that our algorithms can be easily transformed to achieve similar performance, bigger by factor $O(\log n)$, in the latter model, with high probability (i.e., with probability at least $1 - n^{-c}$, for some suitable constant $c > 1$). Another useful property that could be almost immediately derived from this transformation is that nodes do not need to know their exact positions, but only their estimates — this inaccuracy could be overcome by setting a slightly smaller deviation parameter η of the stochastic distribution of random disturbances (although this may in turn result in increasing the error probability ζ of deviating SINR by factor outside of the range $(1 - \eta, 1 + \eta)$, the asymptotic performances would still remain the same with respect to parameters n, D, g).

Our approach is based on propagating the source message to locally and online elected leaders of nearby boxes first, and then to the remaining nodes in those boxes. The main challenge in this process is the lack of knowledge about neighbor location. We solve it through a cascade of diluted transmissions, each initiated by already elected nearby temporary leaders who try to eliminate other leaders in close proximity. This size of this proximity is exponentially increasing in the cascade of these elimination processes, so that at the end only a few nearby leaders in reasonably large distance (to assure a long “hop” of the source message) survive and are used as relays. In case the network granularity is unknown, strongly selective families of specifically selected parameters are used in elimination process. Subtle technical issues need to be solved to avoid simultaneous transmissions of many nodes in one region, as it not only disturbs local receivers but may also interfere with faraway transmissions (recall that in case of weak devices, it is not possible to guarantee such property, as there is no scalable broadcasting algorithm). Once all local leaders possess the source message, it is simultaneously propagated to their neighbors in boxes in a sequence of diluted transmissions.

2 Model, Notation and Technical Preliminaries

We consider a wireless network of n stations, also called *nodes*, deployed into a two dimensional Euclidean space. Stations communicate by using a (single-frequency) wireless channel. They have unique integer IDs in set $[\mathcal{I}]$, where the size of the domain \mathcal{I} is bounded by some polynomial in n . (We use the notation $[i, j] = \{k \in \mathbb{N} \mid i \leq k \leq j\}$ and $[i] = [1, i]$, for any two positive integers i, j .) Stations are located on the plane with *Euclidean metric* $\text{dist}(\cdot, \cdot)$, and each station knows its Euclidean coordinates. Each station v has its *fixed transmission power* P_v , which is a positive real number; whenever station v chooses to transmit a message, it uses its full transmission power P_v . In this work we consider a uniform transmission power setting in which $P_v = P$, for some fixed $P > 0$ and every station v . There are three fixed model parameters, related to the physical nature of wireless medium and devices: path loss $\alpha > 2$, threshold $\beta \geq 1$, and ambient noise $\mathcal{N} \geq 1$. The $\text{SINR}(v, u, \mathcal{T})$ ratio, for given stations u, v and a set of (transmitting) stations \mathcal{T} , is defined as follows:

$$\text{SINR}(v, u, \mathcal{T}) = \frac{P_v \text{dist}(v, u)^{-\alpha}}{\mathcal{N} + \sum_{w \in \mathcal{T} \setminus \{v\}} P_w \text{dist}(w, u)^{-\alpha}} \quad (1)$$

In the *classical Signal-to-Interference-and-Noise-Ratio (SINR) model*, station u successfully receives a message from station v in a round if $v \in \mathcal{T}$, $u \notin \mathcal{T}$, and

$$\text{SINR}(v, u, \mathcal{T}) \geq \beta,$$

where \mathcal{T} is the set of stations transmitting at that round.

However, in practice the above SINR-based condition is too simplistic to capture the complexity of the environment, especially in case of ad hoc networks [20]. In this work, we consider two enhanced versions of the classical SINR model, well-established in the literature: the SINR model with opportunistic links, and the SINR model with random disturbances. In the former model, the SINR ratio is used to decide about successful message delivery, however some links (between faraway nodes) are not taken into account in progress analysis. In the latter model, each SINR ratio is modified by some random factor. In this work we consider a general setting with no restrictions on independence of these random disturbances over nodes, nor their specific distributions. The only two assumptions made are: (i) each random factor is in the interval $(1 - \eta, 1 + \eta)$ with probability at least $1 - \zeta$, for some constant parameters $\eta, \zeta \in (0, 1)$, and (ii) for any given pair of nodes, disturbances of the SINR for these two nodes are independent over rounds.

In order to specify the details of broadcasting task and performance analysis in both models, we first need to introduce the notion of transmission ranges and communication graphs.

Ranges and uniformity. The *communication range* r_v of a station v is the radius of the ball in which a message transmitted by the station is heard, provided no other station transmits at the same time. That is,

r_v is the largest value such that $SINR(v, u, \mathcal{T}) \geq \beta$, provided $\mathcal{T} = \{v\}$ and $d(v, u) = r_v$. As mentioned before, in this paper we consider uniform networks, i.e., when ranges of all stations are equal (to some constant P). Thus, $r_v = r$ for $r = \left(\frac{P}{\beta N}\right)^{1/\alpha}$ and each station v . For simplicity of presentation and wlog, we assume that $r = 1$, which implies that $P = \beta N$. The assumption that $r = 1$ can be dropped without changing asymptotic formulas for presented algorithms and lower bounds.

Communication graph and graph notation. The *communication graph* $G(V, E)$ of a given network consists of all network nodes and edges (v, u) such that $d(v, u) \leq (1 - \varepsilon)r = 1 - \varepsilon$, where $\varepsilon < 1$ is a fixed model parameter. The meaning of the communication graph is as follows: even though the idealistic communication range is r , it may be reached only in a very unrealistic case of single transmission in the whole network. In practice, however, many nodes located in different parts of the network often transmit simultaneously, and therefore it is reasonable to assume that we may hope for a slightly smaller range to be achieved. The communication graph envisions the network of such “reasonable reachability”. Observe that the communication graph is symmetric for uniform networks, which are considered in this paper. By a *neighborhood* of a node u we mean the set (and positions) of all neighbors of u in the communication graph $G(V, E)$ of the underlying network, i.e., the set $\{w \mid (w, u) \in E\}$. The *graph distance* from v to w is equal to the length of a shortest path from v to w in the communication graph, where the length of a path is equal to the number of its edges. The *eccentricity* of a node is the maximum graph distance from this node to all other nodes (note that the eccentricity is of order of the diameter if the communication graph is symmetric — this is also the case in this work).

We say that a station v transmits *c-successfully* in a round t if v transmits a message in round t and this message is received by each station u in Euclidean distance from v smaller or equal to c . We say that node v transmits *successfully* to node u in a round t if v transmits a message in round t and u receives this message. A station v transmits *successfully* in round t if it transmits successfully to each of its neighbors in the communication graph.

Synchronization and rounds. It is assumed that algorithms work synchronously in time slots, also called *rounds*: each station can act either as a sender or as a receiver during a round. We do not assume global clock ticking; algorithm could easily synchronize their rounds by updating round counter and passing it along the network with messages.

Collision detection. We consider the model *without collision detection*, that is, if a station u does not receive a message in a round t , it has no information whether any other station was transmitting in that round, and no information about the received signal, e.g., no information about the value of $SINR(v, u, \mathcal{T})$, for any station v , where \mathcal{T} is the set of transmitting stations in round t .

Broadcast problem and performance parameters. In the broadcast problem studied in this work, there is one distinguished node, called a *source*, which initially holds a piece of information, also called a *source message* or a *broadcast message*. The goal is to disseminate this message to all other nodes by sending messages along the network. Detail performance specification depends on the considered model.

Broadcast in the SINR model with opportunistic links: The complexity measure is the worst-case time to accomplish the broadcast task, taken over all networks with specified parameters that have their communication graphs for fixed model parameter $\varepsilon \in (0, 1)$ connected.

Broadcast in the SINR model with random disturbances: The complexity measure is the worst-case time to accomplish the broadcast task, taken over all networks with specified parameters that have their communication graphs defined for $\varepsilon = 1$ connected. Note that in this model ε is not used as a model parameter, but only with the fixed value 1 to specify the range of admissible networks. Intuitively, the admissible networks in this case are those connected according to “average links”, that is, links for which the expected transmission ranges (i.e., based on expected modified SINR) are taken into account. Observe that the broadcasting time is a random variable, even for deterministic algorithms, due to random disturbances incurred by the model.

Time, also called the *round complexity*, denotes here the number of communication rounds in the execu-

tion of a protocol: from the round when the source is activated with its broadcast message till the broadcast task is accomplished (and each station is aware that its activity in the algorithm is finished). For the sake of complexity formulas, we consider the following parameters: n , \mathcal{I} , D , and g , where n is the number of nodes, $[\mathcal{I}]$ is the domain of IDs, D is the eccentricity of the source, and g is the granularity of the network, defined as r times the inverse of the minimum distance between any two stations (c.f., [6]).

Messages and initialization of stations other than source. We assume that a single message sent in the execution of any algorithm can carry the broadcast message and at most polynomial in the size of the network n number of control bits in the size of the network (however, our randomized algorithms need only logarithmic number of control bits). For simplicity of analysis, we assume that every message sent during the execution of our broadcast protocols contains the broadcast message; in practice, further optimization of a message content could be done in order to reduce the total number of transmitted bits in real executions. A station other than the source starts executing the broadcasting protocol after the first successful receipt of the broadcast message; we call it a *non-spontaneous wake-up model*, to distinguish from other possible settings, not considered in this work, where stations could be allowed to do some pre-processing (including sending/receiving messages) prior receiving the broadcast message for the first time. We say that a station that received the broadcast message is *informed*.

Knowledge of stations. Each station knows its own ID, location coordinates, and parameters n , \mathcal{I} . (However, in randomized solutions, IDs can be chosen randomly from the polynomial range such that each ID is unique with high probability.) Some subroutines use the granularity g as a parameter, though our main algorithms can use these subroutines without being aware of the actual granularity of the input network. We consider two settings: one with local knowledge of density, in which each station knows also the number of other stations in its close proximity (dependent on the ε parameter) and the other when no extra knowledge is assumed.

2.1 Grids

Throughout the paper, we use notation \mathbb{N} for the set of natural numbers, \mathbb{N}_+ for the set $\mathbb{N} \setminus \{0\}$, and \mathbb{Z} for the set of integers. Given a parameter $c > 0$, we define a partition of the 2-dimensional space into square boxes of size $c \times c$ by the grid G_c , in such a way that: all boxes are aligned with the coordinate axes, point $(0, 0)$ is a grid point, each box includes its left side without the top endpoint and its bottom side without the right endpoint and does not include its right and top sides. We say that (i, j) are the coordinates of the box with its bottom left corner located at $(c \cdot i, c \cdot j)$, for $i, j \in \mathbb{Z}$. A box with coordinates $(i, j) \in \mathbb{Z}^2$ is denoted $C_c(i, j)$ or $C(i, j)$ when the side of a grid is clear from the context.

Let ε be the parameter defining the communication graph. Then, $z = (1 - \varepsilon)r/\sqrt{2}$ is the largest value such that the each two stations located in the same box of the grid G_z are connected in the communication graph. Let $\varepsilon' = \varepsilon/3$, $r' = (1 - \varepsilon')r = 1 - \varepsilon'$ and $\gamma' = r'/\sqrt{2}$. We call $G_{\gamma'}$ the *pivotal grid*, borrowing terminology from radio networks research [5].

Boxes $C(i, j)$ and $C'(i', j')$ are *adjacent* if $|i - i'| \leq 1$ and $|j - j'| \leq 1$ (see Figure 1). For a station v located in position (x, y) on the plane we define its *grid coordinates* $G_c(v)$ with respect to the grid G_c as the pair of integers (i, j) such that the point (x, y) is located in the box $C_c(i, j)$ of the grid G_c (i.e., $ic \leq x < (i + 1)c$ and $jc \leq y < (j + 1)c$). The distance between two different boxes is the maximum Euclidean distance between any two points of these boxes; the distance between a box and itself is 0.

A set of stations A on the plane is *d-diluted* wrt G_c , for $d \in \mathbb{N} \setminus \{0\}$, if for any two stations $v_1, v_2 \in A$ with grid coordinates (i_1, j_1) and (i_2, j_2) , respectively, the relationships $(|i_1 - i_2| \bmod d) = 0$ and $(|j_1 - j_2| \bmod d) = 0$ hold.

3 Leader Election in Boxes

The main goal of this paper is to develop two deterministic algorithms: one depending on the knowledge of network granularity, and one general algorithm which does not need such knowledge. The key ingredient

C_1	C_2	C_3
C_8	C	C_4
C_7	C_6	C_5

Figure 1: The boxes C_1, \dots, C_8 are adjacent to C .

of both protocols is a leader election sub-routine. We consider leader election problem defined as follows. Given $x \leq (1 - \lambda)/\sqrt{2}$, for $0 < \lambda < 1$, and a set of “active” stations V , the goal is to choose a leader in each box of the grid G_x containing at least one element of V . In this section we design two algorithms for the defined leader election problem, and in the next section we will show how to apply them to obtain scalable deterministic distributed broadcasting protocols. In every deterministic algorithm, we assume that each message carries all values stored in its sender.

3.1 Granularity-dependent leader election

Let $\text{DilutedTransmit}(V, x, d)$ be the following procedure, consisting of d^2 communication rounds:

Algorithm 1 $\text{DilutedTransmit}(V, x, d)$

- 1: **for** each $a, b \in [0, d - 1]^2$ **do**
 - 2: $A \leftarrow \{v \in V \mid G_x(v) \equiv (a, b) \pmod{d}\}$
 - 3: All elements of A transmit a message
-

Below are two useful properties of DilutedTransmit ; see Appendix for the proof of Proposition 1. We say that a function $d_\alpha : \mathbb{N} \rightarrow \mathbb{N}$ is *flat* if $d_\alpha(n) = O(1)$ for $\alpha > 2$.

Proposition 1. *Let V be a set of at most n stations such that there is at most one station in each box of G_x and $x \leq (1 - \lambda)/\sqrt{2}$ for $0 < \lambda < 1$. Then, there exists a flat function $d_\alpha(n)$ such that each element of V transmits $(2\sqrt{2}x)$ -successfully during $\text{DilutedTransmit}(V, x, \sqrt{d_\alpha(n)})$.*

We say that a box C of the grid G_x has a *leader* from set A if there is one station $v \in A$ located in C with status *leader* and all stations from A located in C know which station it is.

Proposition 2. *Assume that A is a set of leaders in some boxes of the grid G_x , where $x \leq \frac{1-\lambda}{2\sqrt{2}}$, and each station knows whether it belongs to A . Then, it is possible to choose the leader of each box of G_{2x} containing at least one element of A in $O(\frac{d_\alpha(n)}{\lambda})$ rounds, where d_α is a flat function.*

Proof. Note that each box of G_{2x} consists of four boxes of G_x . Let us fix some labeling of this four boxes by the numbers $\{1, 2, 3, 4\}$, the same in each box of G_{2x} . Now, assign to each station from A the label $l \in \{1, 2, 3, 4\}$ corresponding to its position in the box of G_{2x} containing it. We “elect” leaders in G_{2x} in four phases F_1, \dots, F_4 . Phase F_i is just the execution of $\text{DilutedTransmit}(A, x, d)$ for $d = (d_\alpha(n)/\lambda)^{1/\alpha}$ and A equal to the set of leaders with label i (see Proposition 3 in the Appendix). Therefore, each leader from A can hear messages of all other (at most) three leaders located in the same box of G_{2x} . Then, for a box C of G_{2x} , the leader with the smallest label (if any) among leaders of the four sub-boxes of C becomes the leader of C . Finally, complexity bound stated in the proposition follows directly from Proposition 3 in the Appendix and inequality $\alpha > 2$. \square

Algorithms LeadIncrease and GranLeaderElection. Let $\text{LeadIncrease}(A, x, \lambda)$ denote a procedure, which, given leaders of boxes of G_x , chooses leaders of boxes of G_{2x} in $O(\frac{d_\alpha(n)}{\lambda})$ rounds. Such a procedure exists by Proposition 2. Repeating this procedure sufficiently many times for different sets of input parameters, we obtain the following granularity-dependent leader election algorithm.

Algorithm 2 GranLeaderElection(V, g, z)

```
1:  $x \leftarrow \max\{\frac{z}{2^i} \mid i \in \mathbb{N}, \frac{z}{2^i} \leq \frac{1}{g}\}$ 
2:  $A \leftarrow V$  ▷ Each station is a leader of its box of  $G_x$ 
3:  $d \leftarrow d_\alpha(n)$ 
4: while  $x \leq z/2$  do
5:    $\lambda \leftarrow (1 - 2\sqrt{2}x)$ 
6:   LeadIncrease( $A, x, \lambda$ )
7:    $A \leftarrow$  leaders of boxes of  $G_{2x}$ 
8:    $x \leftarrow 2x$ 
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Let $d_\alpha(n)$ be a flat function from Proposition 2.

Theorem 1. *Given $z < 1/\sqrt{2}$, the algorithm GranLeaderElection(V, g, z) chooses a leader in each box of the grid G_z containing at least one element of V in $O((1/\lambda + \log(gz))d_\alpha(n))$ rounds, where $\lambda = 1 - \sqrt{2}z$ and granularity of a network is at most g .*

Proof. Correctness of GranLeaderElection follows from properties of LeadIncrease and choice of parameters (see Proposition 2). Proposition 2 and the choice of x in line 1 of GranLeaderElection directly imply the bound $O(\frac{d_\alpha(n) \log(gz)}{\lambda})$. However, all but the last execution of LeadIncrease is called with $\lambda \geq 1/2$ which gives the result. \square

3.2 General leader election

In the following, we describe leader election algorithm that chooses leaders of boxes of the grid G_z in $O(\log^2 n / \lambda^2)$ rounds, provided $z < 1/\sqrt{2}$, $\alpha > 2$ and $\lambda = 1 - \sqrt{2}z$.

For a family $\mathcal{F} = (F_0, \dots, F_{k-1})$ of subsets of $[Z]$, an *execution* of \mathcal{F} on a set of stations V is a protocol in which $v \in V$ transmits in round $i \in [0, t-1]$ iff $v \in F_i \bmod k$. A family S of subsets of $[Z]$ is a (\mathcal{I}, k) -ssf (strongly-selective family) if, for every non empty subset Z of $[Z]$ such that $|Z| \leq k$ and for every element $z \in Z$, there is a set S_i in S such that $S_i \cap Z = \{z\}$. It is known that there exists (\mathcal{I}, k) -ssf of size $O(k^2 \log \mathcal{I})$ for every $k \leq \mathcal{I}$, c.f., [2].

In the algorithm choosing leaders of boxes of G_z for $z = (1 - \lambda)/\sqrt{2}$, we use a (\mathcal{I}, k) -ssf family S of size $s = O(\log \mathcal{I})$, where k is a constant depending on λ and on $\alpha > 2$. We will execute S on various sets of stations. The set X_v , for a given execution of S and station v , is defined as the set of IDs of stations belonging to $\text{box}_z(v)$ such that v can hear them during that execution of S .

We provide the pseudo-code of the leader election algorithm in Algorithm 3, and then its correctness and complexity analysis will proceed (some technical details are deferred to the appendix). All references to boxes in the algorithm regard boxes of G_z .

The leader election algorithm GenLeaderElection(V, z) chooses leaders from V in boxes of G_z . It consists of two stages. The first stage gradually eliminates the set of candidates for leaders (simultaneously in all boxes) in consecutive executions of a strongly-selective family S . It is implemented as a for-loop. We call this stage *Elimination*.

Let *block* l of Elimination stage denote the executions of family S for $i = l$. Each “eliminated” station v has assigned the value $ph(v)$, which is equal to the number of the block in which it is eliminated. Let $V(l) = \{v \mid ph(v) > l\}$ and $V_C(l) = \{v \mid ph(v) > l \text{ and } \text{box}_z(v) = C\}$, for $l \in \mathbb{N}$ and C being a box of grid G_z . The key properties of sets $V_C(l)$ are: $|V_C(l+1)| \leq |V_C(l)|/2$ for every box C and $l \in \mathbb{N}$, and the granularity of $V_C(l_C^*)$ is smaller than n/z for every box C and l_C^* being the largest $l \in \mathbb{N}$ such that $V_C(l)$ is not empty. Thus, in particular, $V_C(l) = \emptyset$ for each $l \geq \log n$ and each box C of G_z . Motivated by the above observations, the algorithm in its second stage chooses the leader of each box C by applying — simultaneously in each box — the granularity-dependent leader election algorithm GranLeaderElection on

Algorithm 3 GenLeaderElection(V, z)

```
1: For each  $v \in V$ :  $cand(v) \leftarrow true$ ;
2: for  $i = 1, \dots, \log n + 1$  do ▷ Elimination
3:   for  $j, k \in [0, 2]$  do
4:     Execute  $S$  twice on the set: ▷  $S$  is  $(\mathcal{I}, d)$ -ssf of length  $O(d^2 \log \mathcal{I})$ ,  $d$  large enough [2]
5:      $\{w \in V \mid cand(w) = true, w \in C_z(j', k') \text{ such that } (j', k') \equiv (j, k) \pmod{2}\}$ ;
6:     Each  $w \in V$  determines and stores  $X_w$  during the first execution of  $S$  and
7:      $X_v$ , for each  $v \in X_w$ , during the second execution of  $S$ , where
8:      $X_u$  is the set of nodes from box of  $u$  heard by  $u$  during execution of  $S$  on  $V$ ;
9:   for each  $v \in V$  do
10:     $u \leftarrow \min(X_v)$ 
11:    if  $X_v = \emptyset$  or  $v > \min(X_u \cup \{u\})$  then
12:       $cand(v) \leftarrow false$ ;  $ph(v) \leftarrow i$ 
13: For each  $v \in V$ :  $state(v) \leftarrow active$  ▷ Selection
14: for  $i = \log n, (\log n) - 1, \dots, 2, 1$  do
15:    $A_i \leftarrow \{v \in V \mid ph(v) = i, state(v) = active\}$ 
16:    $V_i \leftarrow \text{GranLeaderElection}(A_i, n/z, z)$ 
17:    $\lambda \leftarrow 1 - \sqrt{2}z$  ▷  $V_i$  – new leaders
18:   For each  $v \in V_i$ :  $state(v) \leftarrow leader$ 
19:   DilutedTransmit( $V_i, z, d$ ) for  $d = (d_\alpha(n)/\lambda)^{1/\alpha}$ 
20:   For each  $v \in V$  which can hear  $u \in \text{box}(v)$  during DilutedTransmit( $V_i, z, d$ ):  $state(v) \leftarrow passive$ 
```

$V_C(\log n)$, $V_C(\log n - 1)$, $V_C(\log n - 2)$ and so on, until each box has its leader elected. The second stage of the algorithm is called *Selection*.

Theorem 2. *Algorithm GenLeaderElection(V, z) chooses a leader in each box of G_z containing at least one element of V in $O(\log^2 n)$ rounds, provided $\alpha > 2$ and $\lambda = 1 - \sqrt{2}z > 0$ are constant.*

4 Broadcasting Algorithms

We first describe a generic algorithm DetBroadcast, which uses leader election protocol in boxes of grid G_z for $z = \varepsilon'/\sqrt{2}$, where $\varepsilon' = \varepsilon/2$, as a subroutine (recall that ε is the constant defining the communication graph). The performance of the algorithm is estimated in two variants: the first in which network granularity is known (and GranLeaderElection is applied), and the second which uses GenLeaderElection and does not depend on network granularity.

Let $\gamma' = (1 - \varepsilon')/(2\sqrt{2})$. At the beginning of the algorithm, all stations except of the source s are in the state asleep (states of stations in broadcasting algorithm are independent of their states during their calls to leader election subroutines). In the first round of DetBroadcast, the source sends a message to all stations in its range area; these stations become active) while the source changes its state to passive. Then, the algorithm works in stages $1, 2, 3, \dots$, where the stage i consists of:

- *one execution* of the leader election procedure GenLeaderElection(V_i, z) or GranLeaderElection(V_i, g, z), where $z = \varepsilon'/\sqrt{2}$ and V_i is the set of station in state active at the beginning of the stage, followed by
- $(\gamma'/\varepsilon')^2$ *applications* of DilutedTransmit($V'_{i,a,b}, \gamma', d$) indexed by pairs $(a, b) \in [0, d - 1]^2$, where V'_i are the leaders of boxes of G_z chosen from V_i , $V'_{i,a,b}$ are elements of V'_i with grid coordinates (with respect to G_z equal to (a, b) modulo d and $d = d_\alpha(n)/(2\gamma')$.

The goal of these “diluted” applications of DilutedTransmit is that leaders of boxes of G_z (acting as leaders of boxes of $G_{\gamma'/2}$) send messages to all neighbors (in the communication graph) of all stations from their boxes of G_z . In order to achieve this goal, it is sufficient that leaders transmit $(1 - \varepsilon')$ -successfully. At the end of stage i , all stations in state *active* become *passive* and all stations in state *asleep*, which received the broadcast message during stage i , change state to *active*.

Below we present a pseudo-code of a stage of the broadcasting algorithm DetBroadcast.

Algorithm 4 StageOfBroadcast	▷ a single stage of algorithm DetBroadcast
1: $\varepsilon' \leftarrow \varepsilon/2; \gamma' \leftarrow 1 - \varepsilon'; z \leftarrow \varepsilon'/\sqrt{2}$	
2: $l \leftarrow \lceil \gamma'/\varepsilon' \rceil$	
3: $V \leftarrow$ stations in state <i>active</i>	
4: Run leader election sub-routine: either GenLeaderElection(V, z) or GranLeaderElection(V, g, z)	
5: $V' \leftarrow$ leaders chosen during the leader election in line 4	
6: for each $(a, b) \in [0, l - 1]^2$ do	
7: $V'_{a,b} = \{v \in V' \mid G_z(v) \equiv (a, b) \pmod{l}\}$	
8: $d \leftarrow (d_\alpha(n)/\varepsilon')^{1/\alpha}$	▷ d_α from Prop. 3
9: DilutedTransmit($V'_{a,b}, (1 - \varepsilon')/(2\sqrt{2}), d$)	
10: for each v : if $state(v) = active$: $state(v) \leftarrow passive$	
11: for each v : if $state(v) = asleep$ and v received the broadcast message: $state(v) \leftarrow active$	

Lemma 1. *Algorithm DetBroadcast accomplishes broadcasting in $O(D)$ stages, provided the leader election sub-routine in line 4 of StageOfBroadcast correctly elects leaders in all boxes of grid G_x .*

Proof. We first formulate an essential fact for correctness of our broadcasting algorithm, which easily follows from the definition of a reachability graph.

Fact 1. *Let $\varepsilon' = \varepsilon/2$ for $\varepsilon < 1$. If a station v from a box C of a grid G_x for $x \leq \varepsilon/(2\sqrt{2})$ transmits a message $(1 - \varepsilon')$ -successfully then its message is received by all neighbors (in the reachability graph) of all stations located in C .*

Each station v which receives the broadcast message for the first time at stage j , changes its state from asleep to active at the end of stage j . Then, at the end of stage $j + 1$, such station v changes its state from active to passive. In each stage, only (and exactly) the stations in state active take part as transmitters in leader election and DilutedTransmit. Fact 1 guarantees that, if a station v is in the state *active* during stage j , then all its neighbors in the reachability graph receive the broadcast message during DilutedTransmit (in line 9 of StageOfBroadcast) in stage j . Therefore all neighbors of v (in the reachability graph) are in the state active in stage $j + 1$ or earlier. This implies that broadcasting is finished after $O(D)$ applications of Algorithm 4. \square

Let DetGenBroadcast and DetGranBroadcast denote the broadcasting algorithm using GenLeaderElection and GranLeaderElection, respectively, in line 4 of StageOfBroadcast. Time performances of these leader election protocols, together with Lemma 1, imply the following results.

Theorem 3. *Algorithm DetGenBroadcast accomplishes broadcast in $O(D \log^2 n)$ rounds, provided $\alpha > 2$, $\varepsilon < 1/2$ are constant.*

Proof. The result holds by applying Lemma 1 together with Theorem 2 regarding performance of algorithm GenLeaderElection used for leader election in line 4 of StageOfBroadcast, and by using the facts that the size of S is $O(\log \mathcal{I})$ and that λ'/ε' is constant. \square

Theorem 4. *Algorithm DetGranBroadcast accomplishes broadcast in $O(D(1/\varepsilon^3 + \log g)d_\alpha(n)) = O(D \log g)$ rounds, for constant parameters $\alpha > 2$ and $\varepsilon < 1/2$.*

Proof. Complexity of GranLeaderElection for $z = \varepsilon'/\sqrt{2}$ is $O(d_\alpha(n) \log g)$, since $1 - \sqrt{2}z$ is larger than $1/3$, see Theorem 1. Then, $l = \Theta(1/\varepsilon)$, and $d = \Theta((d_\alpha(n)/\varepsilon)^{1/\alpha})$. Therefore, the for-loop works in $O((1/\varepsilon)^3 d_\alpha(n))$ rounds. Combining this with Lemma 1 yields the theorem. \square

5 Model with Randomly Disturbed SINR

In this section we show simple modifications of original procedures and algorithms from Sections 3 and 4, and argue that their performance in the model with randomly disturbed SINR is bigger by factor $O(\log_{1/\zeta} n)$ than the performance of the original versions analyzed in the model with opportunistic links in Sections 3 and 4. For simplicity, whenever we discuss original algorithms, they are understood to be analyzed in the opportunistic links model, while with respect to the modified algorithms, we assume that they are studied in the randomly disturbed SINR model.

We emulate each round of the original algorithms by $\tau = \Theta(\log_{1/\zeta} n)$ consecutive rounds, and we call them a *phase*. That is, each round of the original algorithms, which we call *original round*, is replaced by a single phase containing τ rounds. Each node transmitting in an original round transmits in all τ rounds of the corresponding phase. However, the local computation done after receiving the signal from the wireless medium in the original round is done only once in the corresponding phase—after receiving the signal from the wireless medium in the final round τ of the phase.

Note that there are more possibilities of receiving messages in a phase, comparing with the corresponding original round, due to random disturbances of SINR ratios. Therefore, in all phases of the modified protocols, each node ignores all messages successfully received from nodes of distance bigger than $(1 - \varepsilon)r$ from it, where $\varepsilon \in (0, 1)$ is defined in such a way that the randomly modified SINR ratio of two nodes of distance at most $(1 - \varepsilon)r$ is above the threshold β with probability at least ζ (note that ε depends on all parameters $\alpha, \beta, \eta, \zeta$). In fact, in the analysis of the original algorithms in Sections 3 and 4 we measured progress only in terms of such opportunistic transmissions between nodes of distance at most $(1 - \varepsilon)r$ from each other; therefore if we explicitly ignore any other (faraway) transmissions in the modified algorithms, we receive the same feedback (from the wireless channel) in phases as in the corresponding original rounds, with high probability (whp). (The exact probability is at least $1 - n^{-c}$, where $c > 1$ is a constant depending on the constant hidden in “ Θ ” notation in the definition of τ .) This is because of three facts: (a) ignoring messages from nodes of distance bigger than $(1 - \varepsilon)r$ allows to focus on the same neighbors as in the progress analysis of the original protocols; (b) the probability that a node does not receive a message from another node of distance at most $(1 - \varepsilon)r$ from it in any round of a given phase, provided it received it in the corresponding original round, is at most ζ^τ , and (c) sufficiently large parameter τ makes the probability small enough inverse of polynomial in n in order to be able to use union bounds of events over all nodes and rounds when transforming the original analysis.

In the Appendix we show that local computation done by original algorithms can be directly transformed into the modified versions of these algorithms as well; this is because they are designed to assure a high level of knowledge consistency, and because the messages received in the original executions are also received in the executions of modified algorithms, whp (as showed above). Thus, enhancing the results in Theorems 3 and 4 by additional factor $\tau = O(\log n)$ coming from simulating each original round by a phase of τ rounds, we obtain the following result.

Theorem 5. *The modified version of algorithm DetGenBroadcast accomplishes broadcast in $O(D \log^3 n)$ rounds, and algorithm DetGranBroadcast accomplishes broadcast in $O(D \log g \log n)$ rounds, with high probability.*

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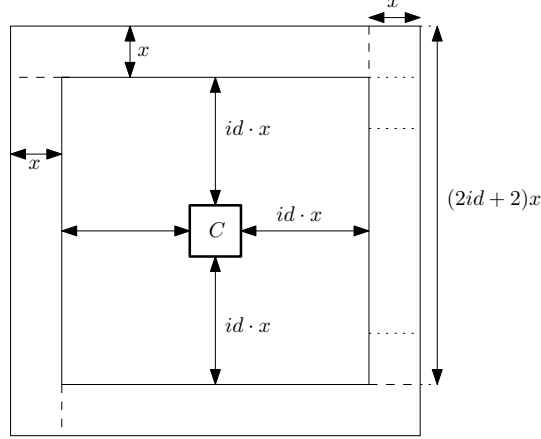


Figure 2: Boxes in distance id from C form a frame partitioned into four rectangles of size $x \times (2id + 2)x$. Each of these rectangles contain at most $i + 1$ boxes such that any two of them are in max-distance at least d .

Appendix

A Useful Properties of Diluted Transmissions and the Proof of Proposition 1

First, we define some useful notation and prove one technical proposition.

Let $I_1 = [i_1, j_1)$, $I_2 = [i_2, j_2)$ be segments on a coordinate axes, whose endpoints belong to the grid G_x . The *max-distance* between I_1 and I_2 with respect to G_x is zero when $I_1 \cap I_2 \neq \emptyset$, and it is equal to $\min(|i_1 - j_2|/x, |i_2 - j_1|/x)$ otherwise. Given two rectangles R_1, R_2 , whose vertices belong to G_x , the max-distance $\text{distM}(R_1, R_2)$ between R_1 and R_2 is equal to the maximum of the max-distances between projections of R_1 and R_2 on the axes defining the first and the second dimension in the Euclidean space.

Proposition 3. *For each $\alpha > 2$ and $\lambda < 1$, there exists a flat function $d_\alpha(n)$ such that the following property holds. Assume that a set of n stations A is d -diluted wrt the grid G_x , where $x \leq (1 - \lambda)/(2\sqrt{2})$ and $d \geq (d_\alpha(n)/\lambda)^{1/\alpha}$. Moreover, $\min_{u,v \in A}(\text{dist}(u, v) \geq \sqrt{2}x)$ (i.e., in particular, at most one station from A is located in each box of G_x). Then, if all stations from A transmit simultaneously, each of them is $2\sqrt{2}x$ -successful. Thus, in particular, each station from a box C of G_x can transmit its message to all its neighbors located in C and in boxes C' of G_x which are adjacent to C .*

Proof. Consider any station u in distance smaller or equal to $2\sqrt{2}x < 3x$ to a station $v \in A$. Then, the signal from v received by u is at least

$$\frac{P}{(2\sqrt{2}x)^\alpha}.$$

Now, we would like to derive an upper bound on interferences caused by stations in $A \setminus \{v\}$ at u . Let C be a box of G_x which contains v . The fact that A is d -diluted wrt G_x implies that the number of boxes containing elements of A which are in max-distance id from C is at most $8(i + 1)$ (see Figure 2). Moreover, no box in distance j from C such that $(j \bmod d \neq 0)$ contains elements of A . Finally, for a station $v \in C$ and a station $w \in C'$ such that $\text{distM}(C, C') = j$, the inequality $\text{dist}(v, w) \geq jx$ is satisfied. Note that our goal is (not) to evaluate interferences (at $v \in C$, but) at any station u such that $\text{dist}(u, v) \leq \frac{2\sqrt{2}x}{c} < 3x$. Therefore, $u \in C'$ such that $\text{distM}(C, C') < 3$, where C' is a box of G_x . For a fixed d , the total noise and interferences

I caused by all elements of $A \setminus \{v\}$ at any location in C is at most

$$\mathcal{N} + \sum_{i=1}^n 8(i+1) \cdot \frac{P}{(i\bar{d}x)^\alpha}$$

Since $\text{distM}(C, C') < 3$ and distM satisfies the triangle inequality, $\text{distM}(C', C'') \geq \text{distM}(C, C'') - 3$ for each C'' , where $u \in C'$. Therefore, if $d > 3$, the noise at u is at most

$$\mathcal{N} + \sum_{i=1}^n 8(i+1) \cdot \frac{P}{(i\bar{d}x)^\alpha}$$

where $\bar{d} = d - 3$. Furthermore,

$$I \leq \mathcal{N} + 8 \cdot \left(\frac{P}{\bar{d}x}\right)^\alpha \cdot \sum_{i=0}^n (i+1)^{1-\alpha} \leq \mathcal{N} + 8e_\alpha(n) \left(\frac{P}{\bar{d}x}\right)^\alpha$$

where $e_\alpha(n) = \sum_{i=1}^n i^{1-\alpha} = 1 + \zeta(\alpha - 1)$, ζ is the Riemann zeta function. So, the signal from v is received at u if the following inequality is satisfied

$$\beta \left(\mathcal{N} + 8e_\alpha(n) \left(\frac{P}{\bar{d}x}\right)^\alpha \right) \leq \left(\frac{P}{2\sqrt{2}x}\right)^\alpha \quad (2)$$

which can be shown equivalent to

$$\bar{d}^\alpha \geq \frac{8 \cdot (2\sqrt{2})^\alpha \beta}{(1 - (2\sqrt{2}x)^\alpha)} \cdot e_\alpha(n)$$

using the assumption $P = \beta\mathcal{N}$. Since $(2\sqrt{2}x) \leq 1 - \lambda$ and therefore $\frac{1}{1 - (2\sqrt{2}x)^\alpha} \leq \frac{1}{\lambda}$, it is sufficient that

$$\bar{d}_\alpha \geq \frac{8(2\sqrt{2})^\alpha \beta}{\lambda} \cdot e_\alpha(n)$$

and, since $\alpha > 2$, that

$$\bar{d} \geq \left(\frac{d_\alpha(n)}{\lambda} \right)^{1/\alpha}$$

where $d_\alpha(n) = 2\sqrt{2} \cdot (8\beta)^{1/\alpha} \cdot (e_\alpha(n))^{1/\alpha}$. □

If the smallest distance between elements of V is larger than or equal to $\sqrt{2}x$ (for $x \leq (1 - \lambda)/(2\sqrt{2})$) then, according to Proposition 3, each station $v \in V$ transmits successfully its message to all its neighbors located in boxes of G_x adjacent to $\text{box}_x(v)$ during execution of $\text{DilutedTransmit}(V, x, d)$ for $d = (d_\alpha(n)/\lambda)^{1/\alpha}$.

Using the above Proposition 3, we can prove Proposition 1 from the main part of the paper.

Proof of Proposition 1: This proposition is a simple corollary from Proposition 3, as $\text{DilutedTransmit}(V, x, d)$ splits V into d -diluted subsets. □

B Deferred Details from the Analysis of GenLeaderElection: Proof of Theorem 2

First, we analyze some properties of communication in the SINR model which eventually will justify application of strongly selective families in GenLeaderElection.

Proposition 4. For each $\alpha > 2$ and $\lambda < 1$, there exists a constant d , which depends only on α , satisfying the following property. Let W be a set of stations such that $\min_{u,v \in W} \{\text{dist}(u, v)\} = \sqrt{2}x \leq (1 - \lambda)$ (i.e., in particular, there is at most one station from W in each box of G_x). Let u, v be the pair of closest stations, i.e., $\text{dist}(u, v) = \sqrt{2}x$ and let $u \in C$, where C is a box of G_x . If u is transmitting in a round t and no other station in any box C' of G_x in the max-distance at most $d/\lambda^{1/(\alpha-2)}$ from C is transmitting at that round, then v can hear the message from u at round t .

Proof. Recall that, according to our assumptions, $P = \beta\mathcal{N}$ and $\sqrt{2}x \leq 1 - \lambda$. The power of signal from u received by v is then

$$S = \frac{P}{(\sqrt{2}x)^\alpha} = \frac{\beta\mathcal{N}}{(\sqrt{2}x)^\alpha}$$

Assuming that no station in any box C' in the max-distance at most d from C is transmitting, the amount of interference and noise at v is at most

$$I \leq \mathcal{N} + \sum_{i=d}^{\infty} 8(i+1) \cdot \frac{1}{(ix)^\alpha} = \mathcal{N} + \frac{8}{x^\alpha} \cdot e_d,$$

where $e_d = \sum_{i=d+1}^{\infty} i^{1-\alpha}$. Thus, it is sufficient to show that $S \geq \beta I$, i.e.,

$$\frac{\beta\mathcal{N}}{(\sqrt{2}x)^\alpha} \geq \beta(\mathcal{N} + \frac{8}{x^\alpha} \cdot e_d)$$

which is equivalent to

$$e_d \leq \frac{\mathcal{N}(1 - (\sqrt{2}x)^\alpha)}{8 \cdot 2^{\alpha/2}}$$

Since $\sqrt{2}x \leq 1 - \lambda$ (and therefore $1 - (\sqrt{2}x)^\alpha \geq 1 - (1 - \lambda)^\alpha \geq \lambda$), the above inequality is satisfied for each e_d such that

$$e_d \leq \frac{\mathcal{N}\lambda}{8 \cdot 2^{1/(2\alpha)}}$$

which holds for sufficiently large d , because of convergence of $\sum_{i=1}^{\infty} 1/i^\alpha$ for $\alpha > 2$.

More precisely, $e_d = \sum_{i=d+1}^{\infty} i^{1-\alpha} \leq \frac{d^{2-\alpha}}{\alpha-2}$ due to the fact that $\sum_{i=d+1}^{\infty} i^{1-\alpha} \leq \int_d^{\infty} i^{1-\alpha}$. Thus, it is sufficient that the following inequality is satisfied

$$d \geq \left(\frac{8 \cdot 2^{\alpha/2}}{\mathcal{N}\lambda(\alpha-2)} \right)^{\frac{1}{\alpha-2}}.$$

□

Corollary 1. For each $\alpha > 2$ and $\lambda < 1/3$, there exists a constant k satisfying the following property. Let W be a set of stations such that $\min_{u,v \in W} \{\text{dist}(u, v)\} = \sqrt{2}x$ and let $\text{dist}(u, v) = \sqrt{2}x$ for some $u, v \in W$. Then, v can hear the message from u during an execution of a (\mathcal{I}, k) -ssf on W .

Proof. Let $d' = \frac{d}{\lambda^{\alpha-2}}$, where d is the constant from Proposition 4. Let $u, v \in W$ be as stated in the corollary and let $l = (2d' + 1)^2$. Conditions of the corollary imply that there is at most one station from W in each box of G_x . Let S be (\mathcal{I}, l) -ssf. Thus, during execution of S for $|S|$ rounds, there exists a round t in which u send a message and no other station in any box at the max-distance at most d' from $\text{box}_x(u)$ sends a message. Proposition 4 implies that v can hear u in round t . □

While Corollary 1 says that a pair of closest station can exchange messages during execution of (\mathcal{I}, k) -ssf, the following proposition generalizes this result by guaranteeing that, simultaneously, the closest pair of stations in each box C can exchange messages, provided there are close enough stations in C .

Proposition 5. *For each $\alpha > 2$ and $\lambda < 1/3$, there exists a constant k satisfying the following property. Let $z \leq (1 - \lambda)/\sqrt{2}$, let W be a d -diluted for $d \geq 3$ wrt G_z set of stations and let C be a box of G_z . Moreover, let $\min_{u,v \in W, \text{box}_z(u) = \text{box}_z(v) = C} \{\text{dist}(u, v)\} = \sqrt{2}x \leq z/n$ and $\text{dist}(u, v) = \sqrt{2}x$ for some $u, v \in W$ such that $\text{box}_z(u) = \text{box}_z(v) = C$. Then, v can hear the message from u during an execution of a (\mathcal{I}, k) -ssf on W .*

Proof. Let u, v and x be as specified in the proposition and let $C = \text{box}_z(u) = \text{box}_z(v)$ be a box of G_z . Let S be a (\mathcal{I}, k) -ssf. If all stations from W are located in C , then the claim follows directly from Corollary 1. So, let W' be the set of all elements of W which are *not* located in C . Let us (conceptually) “move” all stations from W' to boxes adjacent to C , preserving the invariant that $\min_{u,v \in W, \text{box}(u) = \text{box}(v) = C} \{\text{dist}(u, v)\} = x$. Note that such a movement is possible, since there are at most n stations in W' and the side of a box of the grid G_z is larger than $1/2$. Since W is 3-diluted, the distance from $w \in C$ to any station $w' \in W'$ before movement of w' is larger than the distance from w to w' after movement. Let W'' define W with new locations of stations (after movements). Therefore, if u can hear v in the execution of S on W'' (i.e., after movements of stations), it can hear v in the execution of S on W (i.e., with original placements of stations). However, the fact that u can hear v on W'' follows directly from Corollary 1 by the fact that $\min_{u,v \in W''} \{\text{dist}(u, v)\} = x$. \square

Next, we concentrate on properties of Algorithm 3. Recall the notation: $V(l) = \{v \mid \text{ph}(v) > l\}$ and $V_C(l) = \{v \mid \text{ph}(v) > l \text{ and } \text{box}(v) = C\}$, for $l \in \mathbb{N}$ and C being a box of G_z .

Proposition 6. *Let C be a box of G_z for $z \leq (1 - \lambda)/\sqrt{2}$, $\delta < 1/3$ and $l \in \mathbb{N}$. Then,*

- (i) $|V_C(l+1)| \leq |V_C(l)|/2$;
- (ii) *If $V_C(l+1)$ is empty, then the smallest distance between elements of $V_C(l)$ is larger than z/n .*

Proof. Observe that our algorithm implicitly builds matchings in the graph whose nodes are $V_C(l)$ and an edge connects such u and v that u can hear v and v can hear u during an execution of S . In other words, $(u, v) \in E$ when $x \in X_v$ and $v \in X_u$. Moreover, a pair (u, v) belongs to our matching when the following conditions are satisfied:

- $u = \min(X_v)$
- $v = \min(X_u \cup \{u\})$

and therefore also $v < u$. As one can see, each station w can belong to (at most) one such a pair which proves this set of pairs forms a matching in $V_C(l)$ indeed. Note that the station $v \in V_C(l)$ belongs to $V_C(l+1)$ only if it is the smaller element of a pair belonging to our matching. Therefore, the inequality $|V_C(l+1)| \leq |V_C(l)|$ holds. This yields conclusion (i) of the lemma.

As for conclusion (ii), assume that $V_C(l)$ is not empty. Observe that $V_C(l+1)$ is not empty if there exist $v, u \in V_C(l)$ such that v can hear u and u can hear v during execution of S . (Indeed, $v \in V_C(l+1)$ for the smallest $v \in V_C(l)$ such that v can hear u and u can hear v for some $u \in V_C(l)$.) However, such v and u exist if the smallest distance between elements of $V_C(l)$ is at most $\frac{z}{n}$ by Proposition 5. \square

Finally, we are ready to prove Theorem 2.

Proof of Theorem 2: Time complexity $O(\log n \log \mathcal{I})$ follows immediately from the bounds on the size of selectors and complexity of GranLeaderElection.

Proposition 6(i) implies that $V_C(l) = \emptyset$ for each box C and $l > \log n$. (In other words, $\text{ph}(v) \leq \log n$ for each $v \in V$.) For a box C of G_z , let $l_\star = \max_l \{V_C(l) \neq \emptyset\}$. By Proposition 6(ii), the smallest distance between stations of $V_C(l_\star)$ is at least z/n . In other words the smallest distance between stations of

$\{v \in V \mid ph(v) = l_*, state(v) = active\}$ is $\geq z/n$, where l_* is the largest number l such that $ph(v) = l$ for some $v \in V$.

Let us focus on a box C of G_z which contains at least one station from V . Selection stage tries to choose the leader of C among $V_C(\log n), V_C(\log n - 1), \dots$. Moreover, when the leader is elected, all stations from C are switched off (i.e., their state is set to passive which implies that they do not attend further GranLeaderElection executions). Since $l_* = \max_l (V_C(l) \neq \emptyset) \leq \log n$ and the smallest distance between elements of $V_C(l_*)$ is $\geq z/n$, each execution of GranLeaderElection is applied on a set of stations with the smallest distance between stations $\geq z/n$ implying granularity $\Theta(n/z)$, and therefore the leader in each box C containing (at least one) element of V is chosen. \square

C Transforming Algorithms to the SINR Model with Random Disturbances

Below we analyze each type of the original rounds (of algorithms in Sections 3 and 4), and argue that they can be transformed to the model with random disturbances of SINR when applying the general transformation described in Section 5.

Algorithm 1: DilutedTransmit. This procedure does not contain any specific local computation, only transmission pattern (each round of which is simply copied τ times, as specified above in the definition of a phase).

Procedure LeadIncrease. See the proof of Proposition 2 regarding specification of the original procedure LeadIncrease(A, x, λ). In this procedure, leaders of smaller boxes of size x run procedure DilutedTransmit, and at the end the smallest of other at most 3 leaders within the larger box of size $2x$ (containing 4 smaller boxes in total) elects itself as the leader of this box, while the others know it. In the model with randomly disturbed SINR, by the observation above, with high probability each leader of smaller box receives ids of all other (at most 3) leaders of small boxes within the larger box, and therefore all (at most 4) of them select the same leader among themselves using the smallest-id rule as in the original procedure LeadIncrease.

Algorithm 2: GranLeaderElection. This algorithm simply iterates modified procedure LeadIncrease, which gives the same result as the original LeadIncrease, whp, with respect to exponentially growing boxes, and no new local computation rules are used.

Algorithm 3: GenLeaderElection. It contains two parts: elimination and election. In the elimination part, the local computation proceeds with checking conditions (lines 5, and 11) and updating variables (lines 10 and 12). A straightforward inductive argument over the number of runs of the internal part of the loop, lines 4-13, guarantees that, with high probability, sets X_u are the same, and thus the values of variables *cand* and *ph* are the same as in the corresponding original round. Based on them, transmissions are scheduled, which are again the same, whp. In the selection part, only the already computed variables *ph* are taken into account (and we argued that they are the same as in the original execution, whp), and based on them the modified procedures GranLeaderElection and DilutedTransmit are executed, about which we also argued that they yield the same results as the original ones.

Generic algorithm DetBroadcast. It is sufficient to examine the specification of a single stage of the original algorithm, given in Algorithm 4, StageOfBroadcast. In a stage, only a leader election is run once (either GranLeaderElection or GenLeaderElection), which, as we showed, works the same in its original and modified versions, whp. Additionally, common knowledge (known parameters, location) is used, and procedure DilutedTransmit is executed a few times (again, its modified version works the same as the original one, whp). We conclude that both implementations of the generic DetBroadcast algorithms — one based on GranLeaderElection and the other based on GenLeaderElection — work the same in their modified forms as they worked in the original forms, whp.